

Finite $SU(3)^3$ model

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Abstract. We consider $N=1$ supersymmetric gauge theories based on the group $SU(N)_1 \times SU(N)_2 \times \dots \times SU(N)_k$ with matter content $(N, N^*, 1, \dots, 1) + (1, N, N^*, \dots, 1) + \dots + (N^*, 1, 1, \dots, N)$ as candidates for the unification symmetry of all particles. In particular we examine to which extent such theories can become finite, and find that a necessary condition is that there should be exactly three families. From phenomenological considerations an $SU(3)^3$ model is singled out. We consider an all-loop and a two-loop finite model based on this gauge group and we study their predictions concerning the third generation quark masses.

Keywords: Unification of couplings, mass relations; Supersymmetric models

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INTRODUCTION

Finite Unified Theories (FUTs) are $N = 1$ supersymmetric Grand Unified Theories (GUTs) which can be made finite to all-loop orders, including the soft supersymmetry breaking sector. The constructed *finite unified* $N = 1$ supersymmetric $SU(5)$ GUTs predicted correctly from the dimensionless sector (Gauge-Yukawa unification), among others, the top quark mass [1, 2]. Eventually, the full theories can be made all-loop finite and their predictive power is extended to the Higgs sector and the s-spectrum [3]. For a detailed discussion see [4, 5].

Consider a chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g . The superpotential of the theory is given by

$$W = \frac{1}{2} m^{ij} \Phi_i \Phi_j + \frac{1}{6} C^{ijk} \Phi_i \Phi_j \Phi_k, \quad (1)$$

where m^{ij} (the mass terms) and C^{ijk} (the Yukawa couplings) are gauge invariant tensors, and the matter field Φ_i transforms according to the irreducible representation R_i of the gauge group G . All the one-loop β -functions of the theory vanish if the β -function of the gauge coupling $\beta_g^{(1)}$, and the anomalous dimensions of the Yukawa couplings $\gamma_i^{j(1)}$, vanish, i.e.

$$\sum_i \ell(R_i) = 3C_2(G), \quad \frac{1}{2} C_{ipq} C^{jpk} = 2\delta_i^j g^2 C_2(R_i), \quad (2)$$

where $\ell(R_i)$ is the Dynkin index of R_i , and $C_2(G)$ is the quadratic Casimir invariant of the adjoint representation of G . A theorem given in [6, 7, 8, 9] then guarantees the vanishing of the β -functions to all-orders in perturbation theory. This requires that, in

addition to the one-loop finiteness conditions (2), the Yukawa couplings are reduced in favour of the gauge coupling.

In the soft breaking sector, it was found that the soft supersymmetry breaking (SSB) scalar masses in Gauge-Yukawa and finite unified models satisfy a sum rule [10, 11]

$$\frac{(m_i^2 + m_j^2 + m_k^2)}{MM^\dagger} = 1 + \frac{g^2}{16\pi^2} \Delta^{(1)} + O(g^4) \quad (3)$$

for i, j, k , where $\Delta^{(1)}$ is the two-loop correction, which vanishes when all the soft scalar masses are the same at the unification point.

FINITE $SU(3)^3$ MODEL

We now examine the possibility of constructing realistic FUTs based on product gauge groups. Consider an $N = 1$ supersymmetric theory, with gauge group $SU(N)_1 \times SU(N)_2 \times \cdots \times SU(N)_k$, with n_f copies of the supersymmetric multiplet $(N, N^*, 1, \dots, 1) + (1, N, N^*, \dots, 1) + \cdots + (N^*, 1, 1, \dots, N)$. The one-loop β -function coefficient in the renormalization-group equation of each $SU(N)$ gauge coupling is simply given by

$$b = \left(-\frac{11}{3} + \frac{2}{3}\right)N + n_f \left(\frac{2}{3} + \frac{1}{3}\right) \left(\frac{1}{2}\right) 2N = -3N + n_f N. \quad (4)$$

This means that $n_f = 3$ is the only solution of eq. (4) that yields $b = 0$. Since $b = 0$ is a necessary condition for a finite field theory, the existence of three families of quarks and leptons is natural in such models, provided the matter content is exactly as given above.

The model of this type with best phenomenology is the $SU(3)^3$ model discussed in ref. [12], where the details of the model are given. It corresponds to the well-known example of $SU(3)_C \times SU(3)_L \times SU(3)_R$ [13, 14, 15, 16]. Thus, this $SU(3)^3$ model is finite between the Planck M_P and the unification M_{GUT} scales, then breaks spontaneously down to the MSSM at M_{GUT} [15]. Notice that this model has extra exotic particles above M_{GUT} , which are Higgs-like and down-quark like.

With three families, the most general superpotential contains 11 f couplings, and 10 f' couplings, subject to 9 conditions, due to the vanishing of the anomalous dimensions of each superfield. The conditions are the following

$$\sum_{j,k} f_{ijk} (f_{ljk})^* + \frac{2}{3} \sum_{j,k} f'_{ijk} (f'_{ljk})^* = \frac{16}{9} g^2 \delta_{il}, \quad (5)$$

We will assume that the below M_{GUT} , we have the usual MSSM, with the two Higgs doublets coupled maximally to the third generation. The remnants of the $SU(3)^3$ FUT are the boundary conditions on the gauge and Yukawa couplings, i.e. eq. (5), the $h = -MC$ relation, and the soft scalar-mass sum rule (3) at M_{GUT} , which, when applied to the present model, takes the form

$$m_{H_u}^2 + m_{t^c}^2 + m_{\bar{q}}^2 = M^2 = m_{H_d}^2 + m_{b^c}^2 + m_{\bar{q}}^2. \quad (6)$$

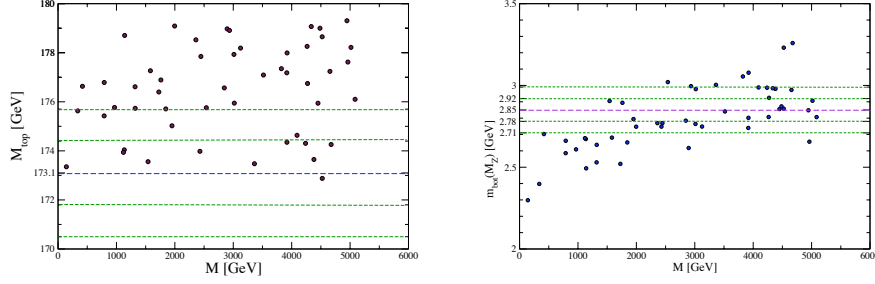


FIGURE 1. The figure shows the values for the top and bottom quark masses for model II, with $r \sim 0.7$ and $\mu < 0$.

Concerning the solution to eq. (5) we consider two versions of the model:

I) An all-loop finite model with a unique and isolated solution, in which f' vanishes, which leads to the following relation

$$f^2 = f_{111}^2 = f_{222}^2 = f_{333}^2 = \frac{16}{9} g^2. \quad (7)$$

As for the lepton masses, because all f' couplings have been fixed to be zero at this order, in principle they would be expected to appear radiatively induced by the scalar lepton masses appearing in the SSB sector of the theory. However, due to the finiteness conditions they cannot appear radiatively and remain as a problem for further study.

II) A two-loop finite solution, in which we keep f' non-vanishing and we use it to introduce the lepton masses. The model in turn becomes finite only up to two-loops since the corresponding solution of eq. (5) is not isolated. In this case we have the following boundary conditions for the Yukawa couplings

$$f^2 = r \left(\frac{16}{9} \right) g^2, \quad f'^2 = (1 - r) \left(\frac{8}{3} \right) g^2. \quad (8)$$

As for the boundary conditions of the soft scalars, we have the universal case.

PREDICTIONS AND CONCLUSIONS

Below M_{GUT} all couplings and masses of the theory run according to the RGEs of the MSSM. Thus we examine the evolution of these parameters according to their RGEs up to two-loops for dimensionless parameters and at one-loop for dimensionful ones imposing the corresponding boundary conditions. We further assume a unique supersymmetry breaking scale M_s and below that scale the effective theory is just the SM.

We compare our predictions with the most recent experimental value $m_t^{exp} = (173.1 \pm 1.3)$ GeV [17], and recall that the theoretical values for m_t suffer from a correction of $\sim 4\%$ [4, 5]. In the case of the bottom quark, we take the value evaluated at M_Z ,

$m_b(M_Z) = 2.85 \pm 0.07 \text{ GeV}$ [18]. In the case of model I, the predictions for the top quark mass (in this case m_b is fixed) m_t are $\sim 183 \text{ GeV}$ for $\mu < 0$, which is above the experimental value.

For the two-loop model II, we look for the values of the parameter r which comply with the experimental limits given above for top and bottom quarks masses. In the case of $\mu > 0$, for the bottom quark, the values of r lie in the range $0.15 \lesssim r \lesssim 0.32$. For the top mass, the range of values for r is $0.35 \lesssim r \lesssim 0.6$. From these values we can see that there is a very small region where both top and bottom quark masses are in the experimental range for the same value of r . In the case of $\mu < 0$ the situation is similar, although slightly better, with the range of values $0.62 \lesssim r \lesssim 0.77$ for the bottom mass, and $0.4 \lesssim r \lesssim 0.62$ for the top quark mass. In this case, if we take some of the exotic particles into account, decoupling a bit below the unification scale, the situation improves. This can be seen in Fig. 1, where we took three down-like and one Higgs-like exotic particles between 10^{15} and 10^{16} GeV , below that the usual MSSM. Then, for $r \sim 0.7$ we have good agreement with experimental data for both top and bottom quarks [19].

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